II.

EXPERIMENTS UPON PIEZOMETERS USED IN HYDRAULIC INVESTIGATIONS.

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In making experiments upon water flowing in pipes and in open conduits, it is most convenient to measure its pressure against the side of the pipe or conduit, and its depth in the conduit, by means of small tubes extending through the side, normal to its surface, and communicating with vertical columns of water, contained in glass tubes or in small reservoirs.

Such columns of water, used to measure pressure, are called piezometers.

The question has arisen: Do they indicate the actual pressure against the side of the pipe, of the water when in motion, or do they indicate the actual height of the surface of the moving water in the conduit?

M. Darcy, in his great work * on the movement of water in pipes, says (page 217): "Indeed, manometers do not indicate the entire head of a conduit at the points where they are adjusted, but this head diminished by a certain height, the diminution being due to the velocity of the fluid at the base of the piezometers: the water, by its cohesion, acts upon the manometric column, whose height it lowers."

Again, he says: "When one of the manometers was placed upon the cylindrical reservoir, where the velocity of the fluid was very slight relatively to that of the water in the pipe, we see that in like circumstances the *lowering by suction* of the manometer upon the reservoir should be less than the *lowering by suction* of the manometer upon the pipe. . . . There was then a rectification to be made, but I have not at this time the means of making it. In the experiments relative to open canals, with which I am now occupied, I shall seek to determine

* Recherches Expérimentales relatives au Mouvement de l'Eau dans les Tuyaux. Par Henry Darcy, Inspecteur-Général des Ponts et Chaussées. Paris, 1857.

the law followed by these lowerings, according to the diameter of the orifice in communication with a current, and according to the velocity of the latter."

In the published records of the experiments relative to open canals,* the results here anticipated are not included, and we are not informed of the later conclusions of this able engineer.

In the performance of my duties as engineer of the Essex Company, — a company controlling the water power of the Merrimack River at Lawrence, Mass., — it has become important to interpret with accuracy the indications of piezometers, and to determine the circumstances affecting their reliability. To this end, a long series of experiments has been made upon piezometers, having orifices of communication of varying size and form, and through a wide range of velocities. The results of these experiments are regarded as of importance to investigators in hydraulics, and are, through the liberality of the officers of the Essex Company, now presented for their use.

It is well known from the experiments of Venturi † that within a short distance from the entrance of a pipe — a distance limited by the influence of the contraction at the entrance — piezometers indicate a pressure varying with their position, and widely different from that which obtains after the section influenced by contraction is passed. These phenomena are readily explained without attributing any discrepancy between the pressure upon the sides of the pipe and the indications of the piezometer. It is now, however, only necessary to consider that portion of the pipe or conduit in which uniform motion is established; that is, a portion in which the particles of water move parallel with the sides of the pipe with a velocity neither increasing nor decreasing.

Uniform motion then existing, the prominent facts to be determined are whether the height of the piezometric column is lowered by the cohesion of the water acting at the base of the piezometer or not; and whether or not the height of the column of still water indicates with accuracy the height of the surface of the adjacent mass of moving water in an open conduit.

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* Recherches Hydrauliques, entreprises par M. H. Darcy, Inspecteur-Général des Ponts et Chaussées: continuées par M. H. Bazin, Ingénieur des Ponts et Chaussées. Première Partie: Recherches Expérimentales sur l'Écoulement de l'Eau dans les Canaux découverts. Paris, 1865.

† Tracts on Hydraulics. Edited by Thomas Tredgold, C. E. II. Venturi's Experiments on the Motion of Fluids. Second Edition. London, 1836, page 136 *et seq.*

For determining these facts, the apparatus represented upon plates No. 1 and No. 2 was constructed. It consisted of a straight trough thirty feet long, of uniform section, one foot deep and three-tenths of a foot wide inside, receiving water at A from a chamber four feet wide. At a distance of six-tenths of a foot up stream from the entrance was a gate B, which, being opened, connected the chamber with a penstock four feet wide, six feet high, and two hundred feet long, bringing water from the Essex Company's south canal. The down-stream side of the chamber A was built up to the height required during the several experiments; and its upper edge used as the crest of a regulating waste weir, over which water continually flowed into the waste trough C, which conducted it outside of the measuring basin.

The experimenting trough discharged its water at D into the swinging conductors, supported by the pivot *a*, which conveyed it by the branch E directly down into the measuring basin G, or by the branch F into the river, as the partition *b* was raised above or lowered below the stream.

The measuring basin G, 15.93 ft. wide, and 36.55 ft. long, and about 8 ft. deep, built of timber and planks on a firm foundation, was buried in earth nearly to its full depth, except on the river side, which was held from being pressed outward by a strong truss; and except at the observer's house H, where the heights of water in the basin were observed.

The area of the measuring basin, within the range of filling during the experiments, which was between 4.5 ft. and 6.5 ft. above the bottom, was, deduction being made for all supporting timbers for the upper works, found to be 570 square feet.

The depths of water in the measuring basin were measured by means of a hook gauge in the box *c*, which was in free communication with different parts of the basin by three pipes, 0.083 ft. in diameter.

The hook gauge used is described and illustrated in "Lowell Hydraulic Experiments,"* page 18.

Water was drawn from the measuring basin through the waste gate *d*.

The experimental trough was at first placed level, having firm bearings about ten feet apart. The upper end was connected with the chamber A with a lining of rubber, making a water-tight joint, which continued water-tight when the other end of trough was lowered

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* Lowell Hydraulic Experiments. By James B. Francis, C. E., &c. Third Edition. New York: D. Van Nostrand. 1871.

through its successive steps to increase the velocity of the water passing through it.

Starting four feet from the entrance, cross bars of wood 0.9 ft. long, 0.1 ft. wide, and 0.15 ft. deep, were screwed to the top of the trough at intervals of just 2.5 ft., the top of the trough being let up into them 0.05 ft.

The up-stream top edge of each cross bar was taken as a station, and these were numbered from 1 to 11, beginning with the up-stream cross bar.

Under the projecting ends of the cross bars were attached, to the outer surface of the two-inch planks which made the sides of the trough, tin boxes about 0.9 ft. long, 0.5 ft. wide, and 0.9 ft. high, having blocks of wood fastened within some of them, as shown upon the plates, to reduce the free surface area of water which they would contain. These boxes, serving as reservoirs, and called still-boxes, were put in communication with the interior of the trough by passages having orifices of various forms and dimensions, and being variously disposed, as expressed in the following table:—

Number of Station.	Side of Trough.	Form of Orifice.	Distance of centre up stream from Station.	Distance of centre above bottom of Trough.	Diameter or Dimensions of Orifice.	Angle of passage with inside of Trough.	Material bordering Orifice.
			Feet.	Feet.	Feet.		
2	West.	Circle.	0.045	0.333	0.043	48° up stream.	Brass.
2	East.	”	0.062	0.333	0.043	48° down stream.	”
3	West.	Circle.	0.057	0.421	0.052	90°	Iron.
3	East.	”	0.054	0.497	0.086	90°	”
4	West.	Circle.	0.059	0.249	0.010	90°	Wood.
4	”	”	”	0.422	0.083	90°	”
4	East.	”	0.058	0.249	0.042	90°	”
4	”	”	”	0.411	0.021	90°	”
5	West.	Rectangle.	0.052	0.505	0.084 wide. 0.337 high.	90°	Wood.
5	East.	”	0.052	0.500	0.021 wide. 0.336 high.	90°	”
6	West.	Circle.	0.054	0.333	0.021	30° up stream.	Brass.
6	East.	”	0.083	0.333	0.021	30° down stream.	”
7	West.	Ellipse.	0.050	0.417	0.030 long. 0.021 high.	45° up stream.	Wood.
7	East.	”	0.053	0.420	0.030 long. 0.021 high.	45° down stream.	”
8	West.	Circle.	0.052	0.255	0.021	90°	Wood.
	”	”	0.052	0.420	0.042	90°	”
8	East.	”	0.052	0.333	0.010	90°	”
		”	0.052	0.502	0.083	90°	”
9	West.	Square.	0.053	0.500	0.167	90°	Wood.
9	East.	Rectangle.	0.053	0.336	0.334 long. 0.083 high.	90°	”
10	West.	Ellipse.	0.053	0.334	0.042 long. 0.021 high.	30° down stream.	Brass.
10	East.	”	0.054	0.334	0.042 long. 0.021 high.	30° up stream.	”

The water flowed toward the north. Distances indicating position of orifices were measured when the trough was level.

The trough with its appurtenances being in place, the whole was covered with a house about eight feet wide and ten feet high, having windows at the top.

The comparative heights of the water surface in the stream and in the reservoirs adjacent were now to be determined. The first step was to measure the heights of three points at each station — one over the middle of the stream, and one over each of the still-boxes — above a surface of still water. This was done by the aid of three kinds of instruments. The first kind, by which any change in height of the water surface was noted, consisted of a plate of brass placed horizontally, through which projected vertically upward fifty-one steel needles in two rows. The first needle being finished with its point just 0.1 ft. above the bottom of the plate, and the fifty-first having its point 0.11 ft. above the same surface, the intermediate points being separated by equal spaces, were finished to be in the same inclined plane with the extremities; hence each point was 0.0002 ft. higher than the next lower.

Six other needles, rising above these in another row, indicated the position of the points reading two-thousandths.

The second kind of instrument consisted of a rod having a scale divided into hundredths of a foot, sliding along a short standard having a stationary vernier reading to thousandths, by which distances of two ten-thousandths of a foot could be readily distinguished. The rods were held in a vertical position by fitting into frames above each point of observation. They were terminated at the lower end by a long finely pointed needle, which was brought in contact with the water surface.

The third kind of instrument consisted of a vertical micrometer screw, piercing a horizontal iron plate which made a part of its nut, and whose under surface was kept level by a level bulb upon its upper surface. The screw was terminated below with a finely pointed needle, and above, near the head, was supplied with an index, whose position was read upon a circular scale made upon the top of the nut, in which one ten-thousandth of a foot was indicated by the space of about one one-hundredth of a foot.

After determining by these instruments the actual heights above a datum plane of all of the points where observations were to be taken, the same instruments were used upon the same points for determining the heights of the water surfaces, when water was flowing through the trough.

During experiments with mean velocities less than three feet per second, the trough was maintained in its level position, and the height of surface and velocity were regulated by screwing a steel plate to the lower end of the trough at the proper height, thus discharging the water over a weir. With greater velocities, the plate was removed, and the trough was more or less inclined.

During experiments, the measurement of the quantity of water flowing through the trough was continuous, interrupted only by drawing water from the measuring basin.

Generally there were as many as four observers, with their instruments, making simultaneous observations at as many stations, with assistants to record their reading.

Upon experimenting with velocities greater than three feet per second, the disturbance at the entrance was found to continue past Station No. 1, consequently all the observations at this station are omitted.

At stations numbered from two to ten inclusive, 5925 observations were made upon the height of the different water surfaces, with velocities in the trough from about 0.6 ft. to about 9 ft. per second. These observations have been divided into 518 experiments, giving a series of heights in each still-box above the surface of the stream at the respective stations. These experiments have been grouped, by putting together those at each station in which the mean velocity and depth of water in the trough were nearly constant, and taking the mean of the heights of the water in each still-box above the surface of the stream. These mean results for each velocity, together with the depth of water, the number of observations, and number of experiments included in each result, are given in the following tables, and are followed by columns of corrected results, which are described in the headings: —

SUMMARY OF RESULTS AT STATIONS No. 2, No. 3, AND No. 4.

Mean velocity of water passing the Station.	Mean depth of water in the Trough.	Mean of observed heights of the surface of water in the still-boxes above the surface of the middle of the stream.		No. of Obser-vat'ns.	No. of Experi-ments.	Mean of observed heights from which the third and fourth columns were obtained, corrected for the variation in height occurring in the interval of time between consecutive experiments.		Correction for slope of the surface of the stream between the point where height was observed and a point opposite the centre of orifice communicating with Still-box.		Corrected mean height of surface of water in the Still-box above the surface of the middle of the stream opposite the centre of orifice communicating with Still-box.		REMARKS.
		At the West Still-box.	At the East Still-box.			At the West Still-box.	At the East Still-box.	At the West Still-box.	At the East Still-box.	At the West Still-box.	At the East Still-box.	
Results at Station No. 2.												
Ft. per sec.	Feet.	Feet.	Feet.			Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	
0.64	0.7726	— 0.0024	+ 0.0032	60	4	— 0.0025	+ 0.0030	. . . . .	. . . . .	— 0.0025	+ 0.0030	
1.03	0.9163	— 0.0064	+ 0.0099	148	9	— 0.0062	+ 0.0101	— 0.0001	— 0.0001	— 0.0063	+ 0.0100	
1.74	0.7752	— 0.0200	+ 0.0226	108	7	— 0.0199	+ 0.0225	— 0.0002	— 0.0002	— 0.0201	+ 0.0223	
2.70	0.8899	— 0.0411	+ 0.0664	119	11	— 0.0410	+ 0.0667	— 0.0002	— 0.0003	— 0.0412	+ 0.0664	
5.23	0.6129	— 0.1880	+ 0.2489	56	5	— 0.1877	+ 0.2479	— 0.0008	— 0.0010	— 0.1885	+ 0.2469	
6.25	0.6812	— 0.2716	. . . . .	34	3	— 0.2713	. . . . .	— 0.0006	. . . . .	— 0.2719	. . . . .	
6.39	0.4113	. . . . .	+ 0.3444	18	4	. . . . .	+ 0.3442	. . . . .	— 0.0032	. . . . .	+ 0.3410	
7.39	0.7719	— 0.3005	. . . . .	19	3	— 0.3015	. . . . .	— 0.0028	. . . . .	— 0.3043	. . . . .	
8.18	0.8340	— 0.4212	. . . . .	24	5	— 0.4203	. . . . .	— 0.0024	. . . . .	— 0.4227	. . . . .	
The ends of both pipes are smooth and the edges square.												
Results at Station No. 3.												
0.64	0.7688	+ 0.0001	— 0.0005	61	4	0.0000	— 0.0006	. . . . .	. . . . .	0.0000	— 0.0006	Both pipes even with inside of Trough.
1.04	0.9105	— 0.0013	0.0000	90	6	— 0.0013	— 0.0001	. . . . .	. . . . .	— 0.0013	— 0.0001	" " " " "
1.76	0.7636	+ 0.0007	+ 0.0022	141	11	+ 0.0007	+ 0.0024	— 0.0001	— 0.0001	+ 0.0006	+ 0.0023	" " " " "

2.77	0.8732	+ 0.0014	+ 0.0022	200	19	+ 0.0013	+ 0.0022	− 0.0002	− 0.0002	+ 0.0011	+ 0.0020	Both pipes even with inside of Trough.
2.80	0.8798	− 0.0263	− 0.0028	24	2	− 0.0257	− 0.0024	− 0.0002	− 0.0002	− 0.0259	− 0.0026	West pipe projects 0.013 ft. East pipe even.
2.09	0.6722	− 0.0181	. . . . .	21	3	− 0.0185	. . . . .	− 0.0002	. . . . .	− 0.0187	. . . . .	” ” ” ” ” ”
2.41	0.6698	− 0.0224	− 0.0028	31	3	− 0.0207	− 0.0009	− 0.0002	− 0.0002	− 0.0209	− 0.0011	” ” ” ” ” ”
5.49	0.6199	− 0.1018	+ 0.0070	20	2	− 0.1018	+ 0.0070	− 0.0007	− 0.0007	− 0.1025	+ 0.0063	” ” ” ” ” ”
6.18	0.6890	− 0.1483	− 0.0028	42	4	− 0.1494	− 0.0041	− 0.0004	− 0.0005	− 0.1498	− 0.0046	” ” ” ” ” ”
7.65	0.7513	− 0.1791	+ 0.0157	42	3	− 0.1792	+ 0.0144	− 0.0023	− 0.0023	− 0.1815	+ 0.0121	” ” ” ” ” ”
8.35	0.8175	− 0.2178	+ 0.0170	18	3	− 0.2175	+ 0.0144	− 0.0021	− 0.0021	− 0.2196	+ 0.0123	” ” ” ” ” ”
7.97	0.8554	− 0.2097	+ 0.0141	55	6	− 0.2107	+ 0.0123	− 0.0018	− 0.0018	− 0.2125	+ 0.0105	” ” ” ” ” ”
7.95	0.8642	− 0.4379	+ 0.0020	7	2	− 0.4374	+ 0.0027	− 0.0018	− 0.0018	− 0.4392	+ 0.0009	West pipe projects 0.055 ft. ” ”
7.98	0.8610	. . . . .	− 0.1065	24	4	. . . . .	− 0.1062	. . . . .	− 0.0018	. . . . .	− 0.1080	and draws air in. East pipe projects 0.008 ft.
7.86	0.8673	− 0.0697	− 0.1805	30	3	− 0.0683	− 0.1779	− 0.0018	− 0.0018	− 0.0701	− 0.1797	West pipe projects 0.007 ft. East pipe projects 0.012 ft.
7.02	0.5575	− 0.0513	. . . . .	71	11	− 0.0512	. . . . .	− 0.0019	. . . . .	− 0.0531	. . . . .	” ” ”

West pipe has its end parallel with side of trough.
East pipe, up-stream edge projects 0.0009 ft. more into trough than down-stream edge, and top projects 0.0002 ft. more than bottom.

Results at Station No. 4.

0.64	0.7646	+ 0.0002	− 0.0004	45	3	+ 0.0002	− 0.0002	. . . . .	. . . . .	+ 0.0002	− 0.0002	Four orifices open.
1.06	0.9078	− 0.0004	− 0.0004	124	7	− 0.0002	− 0.0003	. . . . .	. . . . .	− 0.0002	− 0.0003	” ” ”
1.78	0.7513	− 0.0007	+ 0.0002	169	14	− 0.0008	+ 0.0003	− 0.0001	− 0.0001	− 0.0009	+ 0.0002	0.083 ft. orifice and 0.021 ft. orifice, open.
2.84	0.8498	− 0.0004	+ 0.0008	142	14	− 0.0005	+ 0.0005	− 0.0002	− 0.0002	− 0.0007	+ 0.0003	” ” ” ” ”
5.47	0.6173	+ 0.0020	− 0.0038	47	4	+ 0.0016	− 0.0052	− 0.0009	− 0.0009	+ 0.0007	− 0.0061	” ” ” ” ”
6.17	0.6875	− 0.0069	− 0.0178	45	4	− 0.0080	− 0.0195	− 0.0006	− 0.0006	− 0.0086	− 0.0201	” ” ” ” ”
6.76	0.3843	+ 0.0103	+ 0.0079	20	3	+ 0.0102	+ 0.0073	− 0.0027	− 0.0027	+ 0.0075	+ 0.0046	0.083 ft. orifice and 0.042 ft. orifice open.
7.21	0.5500	+ 0.0033	+ 0.0097	54	6	+ 0.0035	+ 0.0100	− 0.0023	− 0.0023	+ 0.0012	+ 0.0077	0.083 ft. orifice, 0.042 ft. orifice, and 0.021 ft. orifice, open
7.79	0.7379	+ 0.0049	− 0.0155	59	6	+ 0.0049	− 0.0155	− 0.0030	− 0.0030	+ 0.0019	− 0.0185	” ” ” ” ” ” ”
8.04	0.8551	− 0.0021	+ 0.0088	82	9	− 0.0019	+ 0.0093	− 0.0023	− 0.0023	− 0.0042	+ 0.0070	” ” ” ” ” ” ”
8.32	0.8130	− 0.0067	+ 0.0020	18	3	− 0.0052	+ 0.0047	− 0.0026	− 0.0026	− 0.0078	+ 0.0021	” ” ” ” ” ” ”

West side of trough slightly convex towards the middle, ordinate 0.002 ft. in 4 ft. Edge of orifice satisfactorily in surface of trough, but a slight burr at the edge can be drawn out by the finger.

East side of trough is straight, and the edges of orifices are as near in the plane as if in metal.

SUMMARY OF RESULTS AT STATIONS No. 5, No. 6, AND No. 7.

Mean velocity of water passing the Station.	Mean depth of water in the Trough.	Mean of observed heights of the surface of water in the still-boxes above the surface of the middle of the stream.		No. of Observat'ns.	No. of Experiments.	Mean of observed heights from which the third and fourth columns were obtained, corrected for the variation in height occurring in the interval of time between consecutive experiments.		Correction for slope of the surface of the stream between the point where the height was observed and a point opposite the centre of orifice communicating with Still-box.		Corrected mean height of surface of water in the Still-box above the surface of the middle of the stream opposite the centre of orifice communicating with Still-box.		REMARKS.
		At the West Still-box.	At the East Still-box.			At the West Still-box.	At the East Still-box.	At the West Still-box.	At the East Still-box.	At the West Still-box.	At the East Still-box.	
RESULTS AT STATION No. 5.												
Ft. per sec.	Feet.	Feet.	Feet.			Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	
0.63	0.7672	— 0.0005	— 0.0008	60	2	— 0.0005	— 0.0010	. . . . .	. . . . .	— 0.0005	— 0.0010	
1.05	0.9061	0.0000	— 0.0004	88	6	— 0.0001	— 0.0005	. . . . .	. . . . .	— 0.0001	— 0.0005	
1.79	0.7484	— 0.0001	+ 0.0017	133	14	+ 0.0001	+ 0.0019	— 0.0001	— 0.0001	0.0000	+ 0.0018	
2.88	0.8369	0.0000	+ 0.0017	132	12	— 0.0003	+ 0.0013	— 0.0002	— 0.0002	— 0.0005	+ 0.0011	
5.46	0.5883	+ 0.0117	+ 0.0138	57	4	+ 0.0108	+ 0.0116	— 0.0006	— 0.0007	+ 0.0102	+ 0.0109	
6.10	0.7027	+ 0.0038	+ 0.0056	12	1	+ 0.0036	+ 0.0059	— 0.0007	— 0.0007	+ 0.0029	+ 0.0052	
7.05	0.3715	+ 0.0103	+ 0.0146	25	4	+ 0.0100	+ 0.0141	— 0.0022	— 0.0024	+ 0.0078	+ 0.0117	
7.97	0.7114	+ 0.0186	+ 0.0263	35	3	+ 0.0180	+ 0.0255	— 0.0023	— 0.0025	+ 0.0157	+ 0.0230	
8.55	0.8037	+ 0.0215	+ 0.0317	25	4	+ 0.0217	+ 0.0315	— 0.0021	— 0.0023	+ 0.0196	+ 0.0292	

On the West side, the top and bottom of the orifice and the down-stream edge are in the plane of the side of trough, but the up-stream edge recedes 0.0008 ft. at the middle.

On the East side, the plane of the side of trough for a distance of 0.5 ft. up stream from the orifice, continued across, would cut into the down-stream side of orifice 0.0002 ft. for about two-thirds of the lower part of its height.

Results at Station No. 6.												
0.65	0.7647	− 0.0023	+ 0.0018	180	8	− 0.0022	+ 0.0018	. . . . .	. . . . .	− 0.0022	+ 0.0018	
1.06	0.9060	− 0.0043	+ 0.0084	58	3	− 0.0044	+ 0.0083	. . . . .	. . . . .	− 0.0044	+ 0.0083	
1.82	0.7374	− 0.0126	+ 0.0260	102	8	− 0.0126	+ 0.0261	− 0.0001	− 0.0002	− 0.0127	+ 0.0259	
1.89	0.3709	− 0.0122	+ 0.0282	36	4	− 0.0110	+ 0.0298	− 0.0001	− 0.0003	− 0.0111	+ 0.0295	
2.32	0.4736	− 0.0181	+ 0.0413	36	4	− 0.0177	+ 0.0421	− 0.0001	− 0.0003	− 0.0178	+ 0.0418	
2.68	0.6067	− 0.0179	+ 0.0622	18	2	− 0.0169	+ 0.0653	− 0.0001	− 0.0003	− 0.0170	+ 0.0650	
3.01	0.8094	− 0.0327	+ 0.0725	131	12	− 0.0325	+ 0.0727	− 0.0001	− 0.0005	− 0.0326	+ 0.0722	
5.63	0.5929	− 0.1084	+ 0.2472	44	3	− 0.1096	+ 0.2446	− 0.0006	− 0.0013	− 0.1102	+ 0.2433	
6.08	0.7101	− 0.1297	+ 0.2705	30	2	− 0.1296	+ 0.2700	− 0.0007	− 0.0012	− 0.1303	+ 0.2688	
7.11	0.3710	. . . . .	+ 0.3505	11	3	. . . . .	+ 0.3501	. . . . .	− 0.0036	. . . . .	+ 0.3665	
7.54	0.5264	− 0.1734	. . . . .	42	7	− 0.1732	. . . . .	− 0.0028	. . . . .	− 0.1760	. . . . .	**Air draws into the West tube at times.**
8.13	0.7062	− 0.2116	. . . . .	12	2	− 0.2121	. . . . .	− 0.0020	. . . . .	− 0.2141	. . . . .	
8.21	0.8332	− 0.2122	. . . . .	84	12	− 0.2126	. . . . .	− 0.0029	. . . . .	− 0.2155	. . . . .	
8.59	0.8061	− 0.2458	. . . . .	11	3	− 0.2459	. . . . .	− 0.0023	. . . . .	− 0.2482	. . . . .	

On the West side, the plate does not vary more than 0.0003 ft. from the plane of the side of trough.
On the East side, the plate does not vary more than 0.0002 ft. from the plane of the side of trough.

Results at Station No. 7.											
0.65	0.7640	0.0000	+ 0.0003	120	4	− 0.0001	+ 0.0001	. . . . .	. . . . .	− 0.0001	+ 0.0001
1.07	0.9019	+ 0.0003	− 0.0009	77	5	− 0.0002	− 0.0010	. . . . .	. . . . .	− 0.0002	− 0.0010
1.84	0.7313	+ 0.0001	+ 0.0014	162	13	0.0000	+ 0.0014	− 0.0001	− 0.0001	− 0.0001	+ 0.0013
3.11	0.7813	+ 0.0012	+ 0.0037	113	11	+ 0.0010	+ 0.0034	− 0.0001	− 0.0001	+ 0.0009	+ 0.0033
5.58	0.5922	− 0.0049	+ 0.0051	45	4	− 0.0061	+ 0.0040	− 0.0007	− 0.0007	− 0.0068	+ 0.0033
6.08	0.7059	− 0.0067	+ 0.0020	30	3	− 0.0075	+ 0.0013	− 0.0008	− 0.0008	− 0.0083	+ 0.0005
8.24	0.6983	− 0.0137	+ 0.0201	33	3	− 0.0139	+ 0.0203	− 0.0024	− 0.0024	− 0.0163	+ 0.0179
8.65	0.8046	− 0.0332	+ 0.0145	11	2	− 0.0334	+ 0.0145	− 0.0025	− 0.0025	− 0.0359	+ 0.0145

The edges of both orifices are well in the plane of the sides of the trough.
The up-stream edge of the west orifice and the down-stream edge of east orifice are slightly ragged, but do not project inside of trough.

SUMMARY OF RESULTS AT STATIONS No. 8, No. 9, AND No. 10.

Mean velocity of water passing the Station.	Mean depth of water in the Trough.	Mean of observed heights of the surface of water in the still-boxes above the surface of the middle of the stream.		No. of Observat'ns.	No. of Experiments.	Mean of observed heights from which the third and fourth columns were obtained, corrected for the variation in height occurring in the interval of time between consecutive experiments.		Correction for slope of the surface of the stream between the point where the height was observed and a point opposite the centre of orifice communicating with Still-box.		Corrected mean height of the surface of water in the Still-box above the surface of the middle of the stream opposite the centre of orifice communicating with Still-box.		REMARKS.
		At the West Still-box.	At the East Still-box.			At the West Still-box.	At the East Still-box.	At the West Still-box.	At the East Still-box.	At the West Still-box.	At the East Still-box.	
RESULTS AT STATION No. 8.												
Ft. per sec.	Feet.	Feet.	Feet.			Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	
0.65	0.7611	— 0.0007	— 0.0003	93	4	— 0.0007	— 0.0004	. . . . .	. . . . .	— 0.0007	— 0.0004	Four orifices open.
5.76	0.5957	+ 0.0082	+ 0.0053	49	4	+ 0.0077	+ 0.0048	— 0.0006	— 0.0006	+ 0.0071	+ 0.0042	0.042 ft. orifice and 0.083 ft. orifice, open.
7.49	0.3563	+ 0.0131	. . . . .	12	3	+ 0.0125	. . . . .	— 0.0021	. . . . .	+ 0.0104	. . . . .	0.021 ft. orifice in use.
7.76	0.5239	+ 0.0190	+ 0.0048	35	4	+ 0.0194	+ 0.0055	— 0.0018	— 0.0018	+ 0.0176	+ 0.0037	0.021 ft. orifice, 0.042 ft. orifice, and 0.083 ft. orifice, open.
8.36	0.6898	+ 0.0155	+ 0.0072	53	4	+ 0.0149	+ 0.0063	— 0.0023	— 0.0023	+ 0.0126	+ 0.0040	" " " " " " "
8.39	0.8186	+ 0.0164	+ 0.0040	78	9	+ 0.0166	+ 0.0046	— 0.0018	— 0.0018	+ 0.0148	+ 0.0028	" " " " " " "
8.77	0.7826	+ 0.0192	+ 0.0060	17	3	+ 0.0183	+ 0.0044	— 0.0021	— 0.0021	+ 0.0162	+ 0.0023	" " " " " " "

All the edges of orifices are very satisfactorily in the plane of the Trough.

RESULTS AT STATION No. 9.

0.64	0.7650	— 0.0003	— 0.0003	103	5	— 0.0003	— 0.0004	. . . . .	. . . . .	— 0.0003	— 0.0004
1.09	0.8966	— 0.0001	— 0.0009	104	6	0.0000	— 0.0007	. . . . .	. . . . .	0.0000	— 0.0007
1.86	0.7178	— 0.0004	— 0.0002	106	10	— 0.0005	— 0.0004	. . . . .	— 0.0001	— 0.0005	— 0.0005
3.31	0.7322	— 0.0023	— 0.0007	114	11	— 0.0022	— 0.0006	— 0.0001	— 0.0001	— 0.0023	— 0.0007
5.79	0.5702	+ 0.0011	+ 0.0034	47	4	+ 0.0001	+ 0.0017	— 0.0006	— 0.0007	— 0.0005	+ 0.0010
6.14	0.6987	+ 0.0008	+ 0.0046	46	5	— 0.0001	+ 0.0031	— 0.0006	— 0.0007	— 0.0007	+ 0.0024
7.57	0.3474	. . . . .	+ 0.0039	12	3	. . . . .	+ 0.0023	. . . . .	— 0.0021	. . . . .	+ 0.0002
8.44	0.6766	— 0.0041	+ 0.0036	34	3	— 0.0045	+ 0.0032	— 0.0022	— 0.0023	— 0.0067	+ 0.0009
8.88	0.7835	— 0.0054	— 0.0003	17	3	— 0.0032	+ 0.0049	— 0.0021	— 0.0023	— 0.0053	+ 0.0026

On the West side, the lower up-stream corner of orifice projects into the trough 0.0015 ft. The other corners are satisfactorily in the plane of the Trough. On the East side, the edges of the orifice are very satisfactorily in the plane of the Trough.

RESULTS AT STATION No. 10.

0.62	0.7627	0.0000	— 0.0002	141	5	+ 0.0001	— 0.0002	. . . . .	. . . . .	+ 0.0001	— 0.0002
1.08	0.8948	+ 0.0001	— 0.0006	183	9	+ 0.0001	— 0.0006	. . . . .	. . . . .	+ 0.0001	— 0.0006
1.90	0.7081	+ 0.0013	+ 0.0006	157	14	+ 0.0013	+ 0.0007	— 0.0001	— 0.0001	+ 0.0012	+ 0.0006
3.54	0.6901	+ 0.0028	— 0.0020	110	11	+ 0.0027	— 0.0021	— 0.0003	— 0.0003	+ 0.0024	— 0.0024
5.67	0.6639	+ 0.0242	+ 0.0060	50	4	+ 0.0222	+ 0.0022	— 0.0007	— 0.0007	+ 0.0215	+ 0.0015
6.06	0.7089	+ 0.0226	+ 0.0034	46	5	+ 0.0221	+ 0.0026	— 0.0007	— 0.0007	+ 0.0214	+ 0.0019
7.63	0.3462	+ 0.0234	— 0.0007	18	3	+ 0.0229	— 0.0011	— 0.0023	— 0.0023	+ 0.0206	— 0.0034
7.81	0.4893	+ 0.0361	+ 0.0032	27	3	+ 0.0355	+ 0.0024	— 0.0021	— 0.0021	+ 0.0334	+ 0.0003
8.51	0.6783	+ 0.0407	+ 0.0018	31	3	+ 0.0398	+ 0.0007	— 0.0024	— 0.0024	+ 0.0374	— 0.0017
8.75	0.7774	+ 0.0442	— 0.0012	11	2	+ 0.0442	— 0.0012	— 0.0026	— 0 0026	+ 0.0416	— 0.0038

The edges of orifices are satisfactory. Their planes are parallel with sides of trough, and are back from them 0.0003 ft. or 0.0004 ft. on each side.

The following notes were made at Station No. 4, when the mean velocity of the stream was eight feet per second: —

10^h 9′. There is a depression in surface of stream about 0.5 ft. below Sta. No. 3; and a swell at 0.85 ft. above Sta. No. 4. Distance from swell to depression, 1.15 ft.

11′. Depression 0.3 ft. below Sta. No. 4, and swell 1.3 ft. below Sta. No. 4. Distance, 1 ft.

12′. I should judge that the swell above Sta. No. 4 is about 0.02 ft. above the line connecting the depressions.

The depression and swell move longitudinally, frequently about one-half a foot, and more rarely to the extent of replacing each other.

22′. There is continual change in height of surface, but noticeable fluctuations come as often as three-quarters of a second, and from 0.01 ft. to 0.02 ft. in height.

34′. There is a swaying of the highest part of the stream from side to side; generally from within a tenth of a foot from one side to a tenth of a foot from the other, but occasionally running nearer the side.

36′. There are times when the surface is quite even from side to side, and again it will vary as much as 0.02 ft.

38′. A swell longitudinally follows this fluctuation from side to side, but is not a swell the full width of the stream.

39′. The swell seems to be a twisting of the thread of the current from one side of the trough to the other.

41′. In the cross section there is a rise of the surface on each side, and a rise of the thread of the stream for a width of about 0.06 ft., and a depression each side of this.

Immediately after the above observations, the following were made at Station No. 8, in the midst of the series of eleven experiments, having a mean velocity of 8.39 ft. per second.

10^h 46′. The longitudinal distances between the swells and depressions are greater near Sta. No. 8 than at Sta. No. 4, but are less definite. The variation in height is at times as much as 0.02 ft.

52′. The surface of the stream is much more even here than at Sta. No. 4. There is no marked rise near the middle of the stream, but there is a swaying of the highest part of the stream from side to side about 0.01 ft.

54′. The line of air-bubbles is nearly obliterated, varying to 0.04 ft. each side of the middle, but most of the time remaining on the east side of the middle.

Orifices in the Plane of Side. Passages normal.

The results obtained with orifices whose edges do not vary perceptibly from the plane of the side of the trough, with passages normal to this plane will first be considered. Such are found on both sides at Station No. 4, on both sides at Station No. 8, and on the east side at Station No. 9.

Mean velocity of water passing the Stations.		Mean heights of surface of water in the still-boxes above the surface of the middle of the stream, multiplied by the number of observations.										Sum of the products expressed in the preceding columns.	No. of obser-vat'ns.	Average heights, or sum of products divided by number of observations.
		STATION No. 4.				STATION No. 8.				STATION No. 9.				
From.	To.	At the West Still-box.		At the East Still-box.		At the West Still-box.		At the East Still-box.		At the East Still-box.				
Ft. per second.	Ft. per second.	Heights. Feet.	No. of Obs.	Heights. Feet.	No. of Obs.	Heights. Feet.	No. of Obs.	Heights. Feet.	No. of Obs.	Heights. Feet.	No. of Obs.			
0.64	0.65	+ 0.0002 ×	22	— 0.0002 ×	23	— 0.0007 ×	46	— 0.0004 ×	47	— 0.0004 ×	51	— 0.0762	189	— 0.0004
1.06	1.09	— 0.0002 ×	62	— 0.0003 ×	62	. . . .		. . . .		— 0.0007 ×	52	— 0.0674	176	— 0.0004
1.78	1.86	— 0.0009 ×	84	+ 0.0002 ×	85	. . . .		. . . .		— 0.0005 ×	53	— 0.0851	222	— 0.0004
2.84	3.31	— 0.0007 ×	71	+ 0.0003 ×	71	. . . .		. . . .		— 0.0007 ×	57	— 0.0683	199	— 0.0003
5.47	5.79	+ 0.0007 ×	23	— 0.0061 ×	24	+ 0.0071 ×	24	+ 0.0042 ×	25	+ 0.0010 ×	24	+ 0.1691	120	+ 0.0014
6.17	6.14	— 0.0086 ×	22	— 0.0201 ×	23	. . . .		. . . .		+ 0.0024 ×	23	— 0.5963	68	— 0.0088
6.76	7.57	+ 0.0075 ×	7	+ 0.0046 ×	13	+ 0.0104 ×	12	. . . .		+ 0.0002 ×	12	+ 0.2395	44	+ 0.0054
7.21	7.76	+ 0.0012 ×	27	+ 0.0077 ×	27	+ 0.0176 ×	17	+ 0.0037 ×	18	. . . .		+ 0.6061	89	+ 0.0068
7.79	8.44	+ 0.0019 ×	29	— 0.0185 ×	30	+ 0.0126 ×	26	+ 0.0040 ×	27	+ 0.0009 ×	17	— 0.0490	129	— 0.0004
8.04	8.39	— 0.0042 ×	41	+ 0.0070 ×	41	+ 0.0148 ×	39	+ 0.0028 ×	39	. . . .		+ 0.8012	160	+ 0.0050
8.32	8.88	— 0.0078 ×	9	+ 0.0021 ×	9	+ 0.0162 ×	8	+ 0.0023 ×	9	+ 0.0026 ×	9	+ 0.1214	44	+ 0.0028
												+ 0.9950	1440	+ 0.0007

Combining the results obtained at these orifices, grouping those made at the same time or under like circumstances, and giving to each a weight proportioned to the number of observations from which it is obtained, we have the general results contained in the foregoing table.

These average heights are represented in Fig. 1, Plate No. 3. In this Plate and in Plate No. 4, the horizontal lines represent the surface of the stream; the ordinates are the heights expressed in full scale or in actual distances above and below this surface; and the abscissas are mean velocities, in which one foot per second is expressed by one twenty-fourth of a foot.

Taking a general view of these average heights, we find five results are positive and six are negative; and the general average height of all, or the sum of the products of each height multiplied by its number of observations, divided by the whole number of observations, gives for the 1440 observations, the height of the surface of water in the still-boxes, 0.0007 ft. above that of the surface of the middle of the stream.

Examining more in detail, we find that, with mean velocities up to four feet per second, the heights of the surfaces in the still-boxes and those at the middle of the stream coincide, within the practicable limits of measurement. With velocities from five feet to nearly nine feet per second, the heights of surfaces in the still-boxes are both above and below those of the middle of the stream by measurable quantities.

If we now assume, for the purpose of comparison, that, if there is any real difference in these heights dependent upon the velocity, it will vary with the square of the velocity; and assuming also that the incessant fluctuations of the surface of the stream vary in height with the square of the velocity of the stream, it will be found by plotting these heights with their respective velocities, and giving to each a weight corresponding with the number of observations made in determining it, that they will be represented by a line expressing the height of the surface of the still-boxes above that of the stream by 0.000035 of the square of the velocities of the stream; or a little more than two-tenths of one per cent of the heads which would produce these velocities, and but twelve per cent of the extent of the incessant fluctuations in height of the surface of the stream.

This result proves that, with these orifices cut with care in pine planks, having their passages at right angles with the side of the trough, and having their edges so nearly in the plane of the side of

the trough that careful observation detected no variation therefrom, and having areas in circular form from 0.0003 sq. ft. to 0.0054 sq. ft., and one rectangular area having a height of 0.083 ft. and a length of 0.334 ft., there was no lowering of the piezometric column by cohesion of the water acting at its base; and shows that the height of the piezometric column was in excess of that of the stream by an amount extremely small, but with large velocities it was within the practicable limits of observation.

Orifices slightly inclined. Passages normal.

Before concluding the result just obtained to be a general truth, it is important to learn in what manner it will be affected by slight modifications of the conditions under which it was obtained. These are fortunately presented, probably through unequal swelling of the wood, in the orifices on each side at Station No. 5, and at the west orifice at Station No. 9.

After the experiments were made, those with high velocities being recently completed, these orifices were found to be in the following condition: —

At Station No. 5, the orifice upon the west side being a rectangle 0.337 ft. high and 0.084 ft. wide, horizontally, had its top, bottom, and down-stream edges well in the plane of the side of the trough; but the up-stream edge, being in this plane at the top and bottom, receded from it 0.0008 ft. at its mid height. The orifice upon the east side, being a rectangle 0.336 ft. high and 0.021 ft. wide, horizontally, was in the condition that the plane of the side of the trough for half a foot up stream from the orifice continued across it would cut into the down-stream side of the orifice about 0.0002 ft. back from its edge, for about two-thirds of the lower part of its height.

It is probable that these variations from a plane increased during the three months in which the experiments were made, in which case the condition presented is not applicable to the earlier experiments with small velocities.

At Station No. 9, the orifice in the west side was cut 0.168 ft. high and 0.250 ft. long, and then filled for the up stream one-third of its length by a block, leaving a square orifice. At the end of experiments, the edges of the original orifice were found to be in the plane of the side of the trough, and the up-stream and top edges of the block were also in this plane; but the lower down-stream corner of the block, and consequently the lower end of the up-stream edge of the orifice, projected into the trough 0.0015 ft.

With these three orifices, the horizontal elements of the surfaces, bounded by their edges, vary from being parallel with the axis of the stream to a maximum deviation therefrom of 0° 33′ in each of the orifices at Station No. 5, and of 0° 31′ at Station No. 9.

The direction of the deviation of the surfaces being such that at Station No. 5 the particles of water in passing would impinge upon it, and at Station No. 9 they would withdraw from it.

The effect of these slight deviations is presented in the following table, and in Fig. No. 2 of Plate No. 3 : —

Station No. 5. *West Side.*

Mean velocity of water passing the Station. U Ft. per second.	Mean heights of the surface in still-box above that at the middle of the stream. Heights. Feet.	No. of Obs.	$0.0002\ U^2$
0.63	—0.0005	30	0.0001
1.05	—0.0001	44	0.0002
1.79	0.0000	66	0.0006
2.88	—0.0005	61	0.0017
5.46	+0.0102	28	0.0060
6.10	+0.0029	6	0.0074
7.05	+0.0078	12	0.0099
7.97	+0.0157	17	0.0127
8.55	+0.0196	12	0.0150

Station No. 5. *East Side.*

Mean velocity of water passing the Station. U Ft. per second.	Mean heights of the surface in still-box above that at the middle of the stream. Heights. Feet.	No. of Obs.	$0.00032\ U^2$
0.63	—0.0010	30	0.0001
1.05	—0.0005	44	0.0003
1.79	+0.0018	67	0.0010
2.88	+0.0011	61	0.0027
5.46	+0.0109	29	0.0095
6.10	+0.0052	6	0.0119
7.05	+0.0117	13	0.0159
7.97	+0.0230	18	0.0203
8.55	+0.0292	13	0.0240

Station No. 9. *West Side.*

			$0.00008\ U^2$
0.64	—0.0003	51	0.0000
1.09	0.0000	52	0.0001
1.86	—0.0005	53	0.0003
3.31	—0.0023	57	0.0009
5.79	—0.0005	23	0.0027
6.14	—0.0007	23	0.0030
8.44	—0.0067	17	0.0057
8.88	—0.0053	8	0.0063

From these results, we see that, where the surface included by the edges of the orifice is turned, even very slightly, so that the particles of water flowing parallel with the axis of the stream strike into it, the surface of water in the piezometer stands higher than the surface

of the stream; and, when turned so that the particles of water withdraw from it, the surface of the piezometer stands lower than the surface of the stream. Assuming that the variation is as the square of the mean velocity of the stream, the excess in height of the piezometric surface at the west box of Station No. 5 may be expressed approximately by heights equal to $0.0002\,U^2$, in which U stands for the mean velocity of the stream at the station; and the excess in height at the east box of Station No. 5 may be expressed approximately by heights equal to $0.00032\,U^2$; and the depression at Station No. 9 may in like manner be expressed by $0.00008\,U^2$.

Orifices parallel. Passages inclined.

At Stations No. 7 and No. 10, the planes of the edges of the orifices were, within the limits of careful observation, either in the plane of the sides of the trough, as at Station No. 7, or parallel with this plane, and within 0.0004 ft. from it, as at Station No. 10; but at these stations the passages from the orifices, beginning at the plane of their edges, were not normal to this plane, but made therewith an acute horizontal angle.

At Station No. 7, on the west side, the angle was 45° up stream, and on the east side was 45° down stream. At Station No. 10, on the west side, the angle was 30° down stream, and on the east side was 30° up stream.

At Station No. 7, the orifices were made by boring through the sides of the trough, at the proper angle, holes 0.021 ft. in diameter; and, though cut with great care, it was found at the end of the experiments that the acute edge of each was slightly ragged, but no projection into the trough was perceptible.

At Station No. 10, the orifices were made with the same sized hole in plates of brass, carefully finished, and set into the sides of the trough flush; but the swelling of the wood in the thickness of the plate, which was 0.01 ft., caused it to project 0.0003 ft. or 0.0004 ft. beyond the surface of the plates all around. The plates were 0.083 ft. high and 0.125 ft. long.

The results given in the tables for these stations are represented in Fig. No. 3, Plate No. 3.

Here we see that on the east side at Station No. 7, and on the west side at Station No. 10, in which cases the particles of water turning 45° and 30° respectively from their course would flow directly through the passage into the piezometric reservoirs, the surfaces of the reservoirs stand higher than the surface of the stream,

and these heights may be expressed approximately at Station No. 7 by $0.0002\,U^2$, and at Station No. 10 by $0.0005\,U^2$.

On the other hand, on the west side at Station No. 7 and east side at Station No. 10, where the passages go out up stream, the surface of the piezometric reservoir at Station No. 7 stands lower than the surface of the stream by amounts expressed approximately by $0.00025\,U^2$. While at Station No. 10, the surface in the reservoir is slightly above and below that of the stream, in no case more than 0.004 ft., and the mean result for all of the velocities is very nearly zero.

If there were any lowering of the piezometer by the action of cohesion at its base, it would follow that with orifices having passages so very favorable to drawing water from the reservoir, as in the two cases just considered, the lowering would be much greater than the raising above the stream in the two previous cases; but the lowering being really less than the raising tends to the conclusion that there is no lowering due to cohesion at the orifice.

The raising of the piezometric column three per cent of the head that would produce the velocity of the stream, and the lowering of one and one-half per cent of the same head by the difference in direction of the passage, without any perceptible variation of the plane of the orifice from that of the side, indicate that, either from imperceptible variations in the plane of the orifice, or from sinuosity of current, such inclined passages are not to be relied upon for accurate results.

Orifices projecting into the Stream. Passages inclined.

At Station No. 2, a hole was bored through each side of the trough, making with the inner face a horizontal angle of 48°, up stream on the west side and down stream on the east side. Into these holes were fitted brass pipes, 0.049 ft. in diameter outside, and 0.043 ft. in diameter inside, having the inner ends finished smooth with square edges. These pipes projected into the trough, so that the intersection of the plane of the end of each with the plane of the side of the trough was very nearly a tangent to the outer circumference of the end.

The distance from the plane of the side to the point of the outer circumference of the end farthest removed was 0.033 ft., and to the corresponding point of the inner circumference was 0.031 ft.

At Station No. 6 were other projections into the trough. Two brass castings were made, each consisting of a plate 0.083 ft. high,

0.125 ft. long, and 0.010 ft. thick, having near the middle of one side a projection of about 0.02 ft., through which was drilled, lengthwise of the plate at an angle of 30° with its face, a hole 0.021 ft. in diameter. The end of the hole was finished at right angles with its axis, having the circumference very nearly tangent to the surface of the plate; and, the edge of the orifice being as thin as practicable, the outside of the projection was finished, making its elements diverge 10° from the axis of the hole, and its point farthest removed from the plane of the plate was 0.02 ft. therefrom.

When in position in the trough, the face of the plate was in the plane of the side; and a horizontal section through the axis shows the inside of the hole, making an angle of 30°, and the outside of the projection one of 20° with the side. The orifice faced down stream on the west side and up stream on the east side. During experiments, the plates did not vary more than 0.0002 ft. and 0.0003 ft. from the plane of the side of the trough.

A short time before the experiments were completed, an instrument with projections, designed to be miniature models of those at Station No. 6, was set into the west side of the trough, 0.84 ft. up stream from Station No. 6, and 0.25 ft. above the bottom.

This instrument consisted of a circular plate of brass 0.147 ft. in diameter and 0.042 ft. thick, having a hole through its centre 0.013 ft. in diameter, normal to its face, with square edges well in the plane of its face. At 0.04 ft. above and below the central hole were two others, 0.005 ft. in diameter, drilled in horizontal planes through projections upon the face from opposite sides of the vertical through their centres; the hole above the centre making an angle of 30° with the face down stream, and that below making the same angle up stream.

The orifices were finished with thin edges, in planes normal to the axes, and were very nearly tangent to the face of the instrument, from which the entire projection was 0.005 ft. The elements of the outside surfaces were made to diverge 10° from the axes.

When in place in the trough, the face of the instrument was not more than 0.0002 ft. from the plane of the side. Short brass tubes were screwed into the back of the instrument and connected with vertical glass tubes placed against a scale upon which the heights of water surfaces were read.

The results obtained at Stations No. 2 and No. 6 and with the instrument just described are given in the following table: —

Mean velocity of stream passing the orifice. U. Ft. per sc.	Number of observations.	Height of the surface in the still-boxes above that of the middle of the stream. At the West Still-box. Feet.	At the East Still-box. Feet.		
At Station No. 2.				$-0.0063\,U^2$	$+0.0086\,U^2$
0.64	60	— 0.0025	+ 0.0030	— 0.0026	+ 0.0035
1.03	148	— 0 0063	+ 0 0100	— 0.0067	+ 0.0091
1.74	108	— 0.0201	+ 0.0223	— 0.0191	+ 0.0260
2.70	119	— 0.0412	+ 0.0664	— 0.0459	+ 0.0627
5.23	56	— 0.1885	+ 0.2469	— 0.1723	+ 0.2352
6.25	34	— 0.2719	. . . . . .	— 0.2461	. . . . . .
6.39	18	. . . . . .	+ 0.3410	. . . . . .	+ 0.3511
7.39	19	— 0.3043	. . . . . .	— 0.3440	. . . . . .
8.18	24	— 0.4227	. . . . . .	— 0.4215	. . . . . .
At Station No. 6.				$-0.0033\,U^2$	$+0.0074\,U^2$
0.65	180	— 0.0022	+ 0.0018	— 0.0014	+ 0.0031
1.06	58	— 0.0044	+ 0.0083	— 0.0037	+ 0.0081
1.82	102	— 0.0127	+ 0.0259	— 0.0109	+ 0.0245
1.89	36	— 0.0111	+ 0.0295	— 0.0129	+ 0.0264
2.32	36	— 0.0178	+ 0.0418	— 0.0178	+ 0.0398
2.68	18	— 0.0170	+ 0.0650	— 0.0237	+ 0.0536
3.01	131	— 0.0326	+ 0.0722	— 0.0299	+ 0.0670
5.63	44	— 0.1102	+ 0.2433	— 0.1046	+ 0.2346
6.08	30	— 0.1303	+ 0.2688	— 0.1220	+ 0.2736
7.11	11	. . . . . .	+ 0.3465	. . . . . .	+ 0.3741
7.54	42	— 0.1760 *	. . . . . .	— 0.1876	. . . . . .
8.13	12	— 0.2141	. . . . . .	— 0.2181	. . . . . .
8.21	84	— 0.2155	. . . . . .	— 0.2224	. . . . . .
8.59	11	— 0.2482	. . . . . .	— 0 2435	. . . . . .
At Side Instrument above Station No. 6.					
		Height of water above that of central glass tube. In down-stream Tube. Feet.	In up-stream Tube. Feet.	$-0.0025\,U^2$	$+0.0046\,U^2$
7.27	33	— 0.1294	+ 0.2370	— 0.1321	+ 0.2429
7.38	40	— 0.1334	+ 0.2369	— 0.1361	+ 0.2505
8.12	138	— 0.1635	+ 0.3109	— 0.1646	+ 0.3033

* Air drew into West tube at times.

These results are also represented graphically upon Plate No. 4, where it will be seen they may be expressed with a good degree of approximation in terms of the mean velocity of the stream, as follows: —

At Station No. 2.

West side	$-0.0063\ U^2$
East side	$+0.0086\ U^2$

At Station No. 6.

West side	$-0.0033\ U^2$
East side	$+0.0074\ U^2$

At Side Instrument near Station No. 6.

Down-stream Tube	$-0.0025\ U^2$
Up-stream Tube	$+0.0046\ U^2$

In these results, we see that with the orifice inclined down stream the lowering of the reservoir below the surface of the stream is less in amount than the raising of the reservoir above the surface of the stream where the orifice is inclined up stream at the same angle; the former being in the three cases seventy-three, forty-four, and fifty-four per cent of the latter.

Orifices parallel. Passages normal. Tubes projecting.

At Station No. 3, holes were bored at right angles with the plane of the side, and into these were fitted iron pipes. On the west side, the pipe being 0.068 ft. in diameter outside, and 0.052 ft. in diameter inside, had a well finished end parallel with the plane of the side, with square edges. On the east side, the pipe was 0.109 ft. in diameter outside and 0.086 ft. in diameter inside, with end designed to be finished like that upon the west side; but it was not done with care, and the up-stream edge was, at the close of the experiments, found to project 0.0009 ft. farther into the stream than the down-stream edge, and the top edge to project 0.0003 ft. more than the bottom edge.

At first, both pipes were kept flush with, or without any projection beyond the plane of the side, and afterward were pushed out into the stream, as indicated in the table giving a summary of results at this station.

With velocities from 0.64 ft. to 2.77 ft. per sec., the westerly pipe being flush with the side, the average height in the reservoir is the same as that at the middle of the stream, within the practicable limits of measurement.

The same result obtains upon the east side up to velocities exceed-

ing 6 ft. per sec. With velocities between seven and one half feet and eight and one half feet per sec., the average height in the reservoir was greater than that at the middle of the stream by 0.0108 ft.

But, immediately upon projecting one of the pipes into the stream, a marked change is observed: the surface in the reservoir is immediately lowered; with a velocity of 2.77 ft. per sec., the westerly pipe being flush with the side, the surface in the reservoir was 0.0011 ft. higher than that in the stream, but, upon projecting this pipe 0.013 ft. into the stream, the surface in the reservoir immediately lowered to 0.0259 ft. below that of the stream; the mean velocity continuing 2.80 ft. per sec. With the same projection of 0.013 ft., with velocities from 2.09 ft. to 8.35 ft. per sec., the lowering of the surface of the reservoir below that of the stream increased from 0.0187 ft. to 0.2196 ft. in a series expressed approximately by heights equal to 0.0033 U^2, as shown in Fig. No. 4, Plate No. 3.

Increasing the projection to 0.055 ft., the mean velocity being 7.95 ft. per sec., the lowering of the reservoir was 0.4392 ft., or 0.0069 U^2; and it would probably have been more, but at this height the surface of the reservoir was drawn below the top of the pipe, even to its centre, and air was drawn into the stream.

The form of the stream of air at the orifice revealed to the eye the cause of the lowering in the reservoir. It was evident that the particles of water striking the up-stream side of the pipe were deviated in part nearly at a right angle toward the middle of the stream, and their course again bent down stream, forming a path approximating a quadrant of an ellipse whose conjugate axis lay in the end of the pipe parallel with the axis of the stream.

Upon withdrawing the pipe and reducing the projection to 0.007 ft., the lowering in the reservoir is expressed by 0.0011 U^2.

At the east side, with the pipe so placed that its most projecting point was in the plane of the side of the trough, and with a mean velocity of stream of 7.95 ft. per sec., the surface of water in the reservoir was 0.0009 ft. above that of the stream; but, upon projecting the pipe 0.008 ft., the surface in the reservoir was lowered to 0.1080 ft. below that of the stream, the mean velocity continuing 7.98 ft. per sec. This lowering may be expressed by 0.0017 U^2.

Upon projecting this pipe 0.012 ft., the lowering, with a mean velocity of 7.86 ft. per sec., amounted to 0.1797 ft., or 0.0029 U^2.

If we knew the actual distribution of velocities throughout the stream, it would be interesting to trace the relation of the lowering of the reservoir and the velocity of the stream just at the end of the pipe.

With our present knowledge, this cannot be done with accuracy; but from observations, which I will not describe, I am able to present an approximate result which will serve to illustrate one principle.

Taking, for example, the conditions when the mean velocity of the stream is 8 ft. per sec., I construct the following table: —

Distance from side of trough.	Approximate velocity at the distance given.	Head that would produce this approximate velocity.	Lowering of reservoir below surface of stream.	Lowering of reservoir divided by head producing approximate velocity.	Velocity due lowering. $V=\sqrt{2gh}$	Velocity due lowering divided by approximate velocity.
Feet.	Ft. per sec.	Feet.	Feet.		Ft. per sec.	
0.007	5.6	0.488	0.070	0.14	2.12	0.38
0.008	5.8	0.523	0.109	0.21	2.65	0.46
0.012	6.2	0.598	0.186	0.31	3.46	0.56
0.013	6.3	0.617	0.211	0.34	3.68	0.58
0.055	8.0	0.995	0.442	0.44	5.33	0.67

Recurring to the observations when the westerly tube projected 0.055 ft. and the paths of some of the particles were found to deviate about ninety degrees, and, passing through a quadrant of an ellipse, again resume a direction parallel with their former course, it would be reasonable to conclude that, with a projection of a few thousandths of a foot, the deviation of the paths would be less than ninety degrees, and the curve would be flattened, and become more like a segment taken nearer the transverse axis. In this case, the lowering of the reservoir would not bear a constant relation to the head which would produce the velocity existing at the end of the pipe, but would be a fraction of this head, increasing with the distance from the side, rapidly at first, and then slower, until reaching a limit would remain constant. This conclusion is confirmed by the results of the table, in which with distances from 0.007 ft. to 0.055 ft., the velocity due a height equal to the lowering of the reservoir increases from 0.38 to 0.67 of the velocity at the end of the pipe, and this increase is in a rapidly decreasing series.

I have reason to conclude, from experiments made elsewhere, that the lowering of the reservoir depends also upon the thickness of the end of the pipe; for I found by projecting a pipe 0.02 ft. in diameter, having very little thickness at the end, into water flowing in an iron pipe one foot in diameter, with a mean velocity of 9 ft. per sec., a lowering of the piezometric column greater than that above given; namely, for the same distances from the side, the velocity due a height equal to the lowering was from seventy to ninety one-hundredths

of the velocity with which the water approached the end of the pipe, and at greater distances the velocity due the lowering increased until it exceeded the velocity of the approaching water.

Conclusions.

From the 6000 and more observations made at this trough upon the various forms and kinds of orifices, I reach the following general conclusions : —

The first group of experiments shows that with orifices whose edges are in the plane of the side of the conduit, with passages normal to this plane, the surface of water in the piezometers does not stand below the surface of the stream.

On the contrary, the general results at the orifices of this kind indicate, for the higher velocities, an excess of height in the piezometer expressed by 0.000035 U^2.

This is but twelve per cent of the incessant fluctuation of the surface ; but, though a very small quantity, it is, with the higher velocities experimented upon, a measurable one, and its cause is to be sought.

Experiments at Station No. 6 and those with the instrument having projections of 0.005 ft. show that the height above the surface of the stream to which water in the piezometer is forced is greater when the orifice is turned toward the current than the height below the surface to which it is drawn when the orifice is turned so that the stream draws away from it; in these cases nearly twice as great, the angle with the plane of the side and the amount of projection being the same when facing with or against the current. The observations at Station No. 5 and on the west side at Station No. 9 give a similar result.

The edges of the orifices of the first group of experiments which were, within the limits of careful observation, in the plane of the side of the trough, were of course not perfectly in this plane : the probabilities are that they deviated as much from this plane upon the down-stream side as upon the up-stream side, in which case it follows from the experiments just cited that the effect of these imperfections would give an average height of the piezometers greater than the height of the stream. If the comparative value of the excess of and diminution in height of the experiments cited be applied to these results, the average height of piezometers for velocities above 5 ft. per sec. will be reduced to the average height of the stream, within the practicable limits of measurement. This result indicates, with a nearness of approximation unusual in hydraulic investigations, that

with the plane of the orifice accurately in the plane of the side of the conduit the piezometer will indicate the true height of the surface of the stream.

The second group of experiments shows that, with extremely slight variations of the plane of the orifice from the plane of the side, the piezometer indicates a greater height or a less height than the surface of the stream, according as these variations cause the stream to strike into or draw away from the plane of the orifice; and, in connection with the experiments at Station No. 6 and elsewhere upon definitely formed projections, they lead quite definitely to the conclusion that with an orifice whose edges are in the plane of the side, and passage normal thereto, the piezometric column will stand neither above nor below the surface of the stream, but will indicate the true height of this surface.

The third group of experiments in which the plane of the orifice was in, or nearly in, the plane of the side, but the passage from it turned sharply up stream or sharply down stream, shows that, with such arrangement, variations from the plane of the side which would escape careful observation, or slight inclinations of the current, may lead to variations of considerable magnitude in height of the piezometric column above or below that of the surface of the stream, consequently such arrangements are not to be relied upon.

The fourth group of experiments in which the orifice projects into the stream, and the plane of its edges makes a large horizontal angle with the plane of the side, either up stream or down stream, shows that with the same angle and the same projection into the stream the piezometric column connected with the orifice facing up stream stands above the surface of the stream by a much greater amount than the piezometric column connected with the orifice facing down stream stands below the same surface; the latter height being, in the examples before us, from forty-four to seventy-three per cent of the former.

The fifth group of experiments in which pipes at right angles with the current project into the stream, the end of the pipe having square edges, shows that with such an arrangement the piezometric column stands lower than the surface of the stream.

This follows from the fact made evident that the particles of water which would pass where the pipe is are deviated from their course, a part of them moving lengthwise of the pipe, and, being projected in a curve around its end, cause the pressure into the end of the pipe to be diminished below that of the normal pressure upon the sides of the

trough, by an amount which is a varying fraction of the pressure which would produce the velocity with which the water approaches the pipe.

This fraction increases with the projection from the side, from zero to forty-four one-hundredths, in the experiments in this trough; and in the experiments alluded to in a closed conduit under pressure, with a smaller pipe having a thin end, it increased more rapidly with the same distances from the side, and with greater distances increased until the lowering exceeded the height which would produce the velocity of the approaching water.

The lowering of the piezometric column under circumstances like these just presented confirmed Dubuat * in his conclusion that water in motion pressed upon the sides of a conduit with a pressure less than that due its depth, by the whole amount of pressure that would produce its mean velocity, which conclusion Navier † controverted, but which has been presented in works upon hydraulics quite generally until the publication of the results of M. Darcy, which in general confirmed the position of Navier, but left in doubt the indications of the piezometer; but the experiments now presented show that with currents flowing parallel with the side of a straight conduit, with orifices having edges in the plane of the side and with passages normal thereto, there is no lowering of the piezometric column, but that it indicates the true height of the surface of the water in the conduit when in motion, as well as when at rest. And we have a reliable datum plane, to which observations in hydraulics may be referred.

Note. Upon the Limits of Accuracy that may be obtained with Piezometers.

The experiments of the first group were united, because from careful observation, made before any results were computed, the planes of the orifices were regarded as satisfactorily in the plane of the side. No deviation therefrom was perceptible in the light in which they could be seen that enabled me to say they inclined one way or the other. This light was good upon the side of the straight edge presented to the eye; but, looking down into the trough, no light could be seen beyond, and very slight variations could not be detected there, which could be seen if the bearing surfaces were between the eye and the light.

* Principes d'Hydraulique. Par M. Dubuat. Paris, 1816. Art. 439 and 453.
† Architecture Hydraulique. Par Bélidor. Paris, 1819. Page 342.

It may be that variations of 0.0002 ft. were overlooked, though such were detected at Station No. 5.

To obtain as definite an idea as we may of the precision necessary to obtain accurate results with a piezometer, let the results obtained at the several orifices of the first group be worked up separately. The heights of the piezometers above and below the surface of the stream will be expressed approximately as follows: —

STATION No. 4.	West side, — $0.00004\ U^2$	East side, — $0.00006\ U^2$
STATION No. 8.	West side, + $0.00022\ U^2$	East side, + $0.00006\ U^2$
STATION No. 9.	East side, + $0.00002\ U^2$	

Applying as well as we may the results obtained in the second group of experiments, and assuming the variation in height to be proportional to the angle of inclination with the plane of the side, of the horizontal elements of the surface bounded by the edge of the orifice, it will be seen that an extreme variation from the plane of the side in the length of any of the longer of these elements of 0.0004 ft., 0.0002 ft., 0.0003 ft., 0.0002 ft., and 0.0002 ft., respectively at the several orifices, in the order in which they have been named, will account for the several variations in the height of the piezometric columns above or below the surface of the stream; and a less variation from the plane of the side, in the length of any of the shorter horizontal elements of the circular orifices, would serve to account for them.

It will be observed that the heights by the piezometer whose orifice was 0.334 ft. long horizontally indicated more nearly than those with smaller orifices the actual height of the stream.

From these results, it is evident that it is entirely within the practicable limits of construction to make piezometers that will indicate the true height of the stream, within the practicable limits of observation.